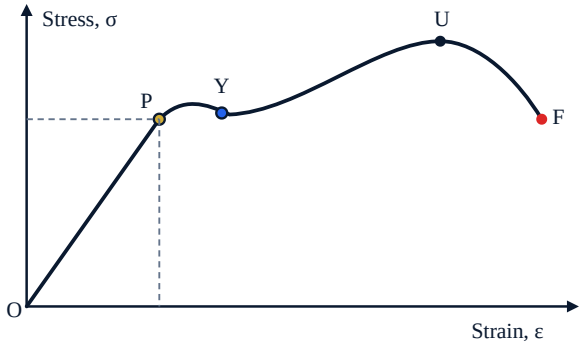
Topic: Materials & Solid Mechanics

BPHO EXTENDED LEVEL

Instructions: Answer all 30 questions. The questions evaluate bulk properties including density, Hooke's law, spring networks, elastic and plastic deformation, tensile stress, strain, strain energy density, and Young modulus. All physical diagrams represent strictly geometrical and experimental fixtures without force vectors. Assume acceleration due to gravity $g = 9.81\ \mathrm{m\ s^{-2}}$ where required.

1 The diagram shows a tensile stress-strain (σ--ε) curve for a structural ductile polycrystalline metal subjected to uniaxial tension until fracture. The line segment OP is strictly linear. Which point denotes the yield point where substantial plastic deformation initiates without a proportional increase in stress?



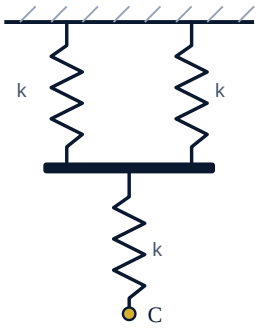
A. Point P (Proportionality Limit)

B. Point Y (Yield Point)

C. Point U (Ultimate Tensile Strength)

D. Point F (Fracture Stress)

2 The diagram depicts a spring system suspended from a rigid horizontal beam. Two identical light springs, each with spring constant k, are connected in parallel. This combination is connected in series with a third identical spring of stiffness k. What is the effective spring constant k_{eff} of the entire arrangement measured at point C?



A. $\frac{1}{3}k$

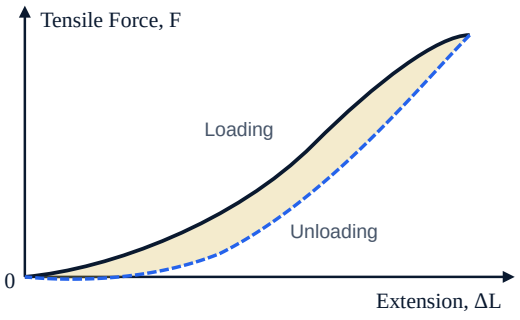
B. $\frac{2}{3}k$

C. $\frac{3}{2}k$

D. $3k$

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3 The graph illustrates a complete loading and unloading cycle for a vulcanized rubber specimen stretched within its elastic limit. What specific physical quantity is represented by the shaded region enclosed between the two curves (the hysteresis loop)?



A. The maximum elastic strain energy stored in the rubber at peak extension.

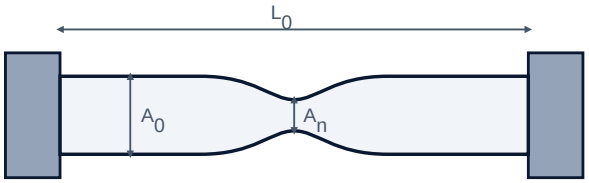
B. The thermal energy dissipated per cycle due to internal molecular friction.

C. The work recovered by the external surroundings during elastic recoil.

D. The plastic work permanently deforming the crystal lattice.

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4 The diagram shows a tensile test specimen of initial uniform circular cross-section A_0 and original gauge length L_0. Under sustained tensile displacement, local plastic instability creates a pronounced reduction in cross-section (necking) at the center to a minimum area $A_n < A_0$. Which statement correctly compares the engineering stress σ_{eng} and the true stress σ_{true} at this instant within the necked region?



A. $\sigma_{\text{true}} = \sigma_{\text{eng}}$ because both are evaluated using the identical instantaneous axial tensile load.

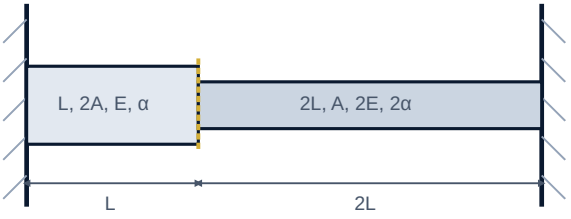
B. $\sigma_{\text{true}} > \sigma_{\text{eng}}$ because σ_{true} accounts for the reduced instantaneous area A_n.

C. $\sigma_{\text{true}} < \sigma_{\text{eng}}$ because plastic yielding relaxes the microscopic internal lattice stress.

D. $\sigma_{\text{true}} = 0$ at the instant necking commences due to stress concentration relief.

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5 The diagram shows a composite rod consisting of two cylindrical metal sections firmly bonded end-to-end between two rigid, immovable walls. Section 1 has length L, cross-sectional area $2A$, and Young modulus E. Section 2 has length $2L$, cross-sectional area A, and Young modulus $2E$. If the temperature of both sections is uniformly raised by ΔT (with linear thermal expansion coefficients $\alpha_1 = \alpha$ and $\alpha_2 = 2\alpha$), what is the magnitude of the compressive force P developed in the composite rod?



A. $P = \frac{1}{3}AE\alpha\Delta T$

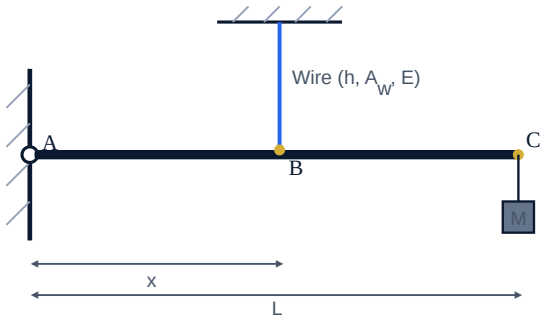
B. $P = \frac{5}{3}AE\alpha\Delta T$

C. $P = 3AE\alpha\Delta T$

D. $P = \frac{10}{3}AE\alpha\Delta T$

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6 The diagram depicts a light, rigid horizontal beam of length L hinged smoothly at wall support A and supported horizontally by a vertical elastic wire at point B (a distance x from A). The wire has length h, cross-sectional area A_w, and Young modulus E. A mass M is suspended from the extreme free end C. Assuming small deflections, what is the downward vertical displacement δ_C of the tip C?



A. $\delta_C = \frac{MghL}{A_w E x}$

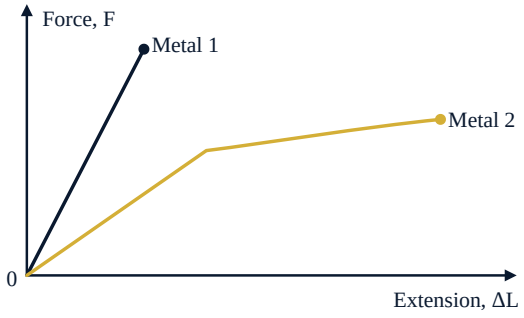
B. $\delta_C = \frac{MghL^2}{A_w E x^2}$

C. $\delta_C = \frac{Mghx^2}{A_w E L^2}$

D. $\delta_C = \frac{Mgh(L-x)}{A_w E x}$

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7 The graph shows the force-extension curves for two distinct structural materials, Metal 1 and Metal 2, tested to fracture with identically dimensioned cylindrical specimens (equal original length L_0 and original cross-sectional area A_0). Which rigorous deduction regarding their bulk mechanical properties is correct?



A. Metal 2 has a higher Young modulus and greater ultimate tensile strength than Metal 1.

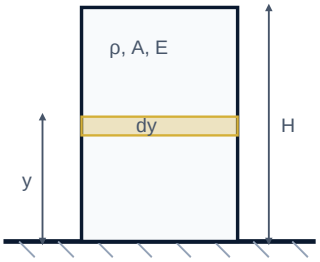
B. Metal 1 has a higher Young modulus and is more brittle than Metal 2.

C. Metal 2 is more brittle and has lower toughness than Metal 1.

D. The elastic strain energy stored at the proportionality limit is strictly identical for both metals.

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8 The diagram depicts a thick uniform vertical column of height H, uniform cross-sectional area A, volumetric mass density ρ, and Young modulus E, resting vertically on a rigid ground support. Considering deformation due strictly to its own self-weight under gravity g, what is the total axial elastic compression ΔH of the column?



A. $\Delta H = \frac{\rho g H^2}{E}$

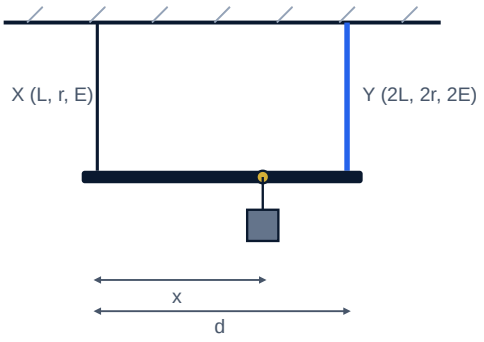
B. $\Delta H = \frac{\rho g H^2}{2E}$

C. $\Delta H = \frac{2\rho g H^2}{E}$

D. $\Delta H = \frac{\rho g A H}{E}$

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9 The diagram shows a light horizontal bracket supported by two parallel vertical wires, X and Y, hanging from a ceiling. Wire X has length L, radius r, and Young modulus E. Wire Y has length $2L$, radius $2r$, and Young modulus $2E$. The distance between the wires is d. At what distance x from wire X must an external load be suspended so that the bracket remains strictly horizontal after deformation?



A. $x = \frac{1}{3}d$

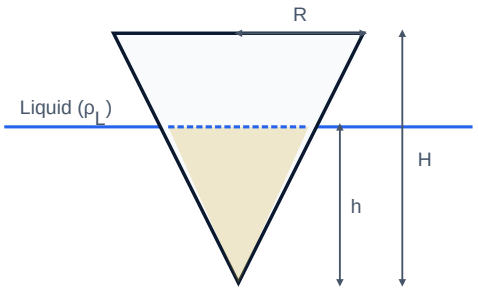
B. $x = \frac{1}{2}d$

C. $x = \frac{2}{3}d$

D. $x = \frac{4}{5}d$

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10 The diagram depicts a solid homogeneous right circular cone of height H, base radius R, and uniform volumetric density ρ_0. The cone floats stably in a liquid of density ρ_L with its sharp vertex pointing downwards. The vertex is submerged to an axial depth h. Which mathematical expression rigorously dictates the ratio $\frac{h}{H}$ at static hydrostatic equilibrium?



A. $\frac{h}{H} = \frac{\rho_0}{\rho_L}$

B. $\frac{h}{H} = \sqrt{\frac{\rho_0}{\rho_L}}$

C. $\frac{h}{H} = \left(\frac{\rho_0}{\rho_L}\right)^{1/3}$

D. $\frac{h}{H} = \left(\frac{\rho_0}{\rho_L}\right)^{3}$

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11 A uniform cylindrical metal wire of length L and cross-sectional area A is stretched elastically by an axial tensile force F, extending by ΔL. If Hooke's law holds throughout, what is the volumetric elastic strain energy density u stored within the material?

A. $u = \frac{1}{2}\sigma\varepsilon$

B. $u = \sigma\varepsilon$

C. $u = \frac{1}{2}E\varepsilon$

D. $u = \frac{F\Delta L}{2L}$

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12 Two wires, P and Q, are manufactured from the identical isotropic alloy. Wire Q has twice the length and three times the diameter of wire P. When subjected to the same tensile tensile load within the linear elastic regime, what is the ratio of the extension of wire Q to that of wire P ($\frac{\Delta L_Q}{\Delta L_P}$)?

A. $\frac{2}{3}$

B. $\frac{2}{9}$

C. $\frac{4}{9}$

D. $\frac{6}{1}$

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13 A spherical solid metal bead of radius R has mass M. It is bored through with a concentric spherical hollow cavity of radius $r = \frac{1}{2}R$. If the remaining metal shell retains mass M, by what factor has the apparent bulk density of the bead decreased compared to the true density of the pure metal?

A. $\frac{1}{8}$

B. $\frac{1}{4}$

C. $\frac{7}{8}$

D. $\frac{8}{7}$

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14 A cylindrical steel specimen with Poisson's ratio $\nu = 0.30$ is subjected to uniaxial tensile stress within its proportional limit, undergoing an axial longitudinal strain of $\varepsilon_{\text{axial}} = +1.0 \times 10^{-3}$. What is the fractional change in the volume of the specimen ($\frac{\Delta V}{V}$)?

A. $+0.40 \times 10^{-3}$

B. $+0.70 \times 10^{-3}$

C. $+1.00 \times 10^{-3}$

D. -0.60×10^{-3}

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15 An unstretched ideal spring of resting length L_0 and spring constant k is cut into two segments of resting lengths $\frac{1}{3}L_0$ and $\frac{2}{3}L_0$. What are the respective spring constants k_1 and k_2 of the shorter and longer segments?

A. $k_1 = \frac{1}{3}k, \quad k_2 = \frac{2}{3}k$

B. $k_1 = 3k, \quad k_2 = \frac{3}{2}k$

C. $k_1 = 3k, \quad k_2 = 2k$

D. $k_1 = \frac{3}{2}k, \quad k_2 = 3k$

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16 A high-strength alloy wire of original cross-sectional area $1.50 \times 10^{-6}\ \text{m}^2$ and original length $2.40\ \text{m}$ has Young modulus $E = 200\ \text{GPa}$. Calculate the elastic strain energy stored in the wire when it is stretched elastically by an axial load of $600\ \text{N}$.

A. $0.72\ \text{J}$

B. $1.44\ \text{J}$

C. $2.88\ \text{J}$

D. $4.80\ \text{J}$

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17 (BPhO Problem) A thin ring of radius R, cross-sectional area A, density ρ, and Young modulus E rotates in a horizontal plane about its central perpendicular symmetry axis with constant high angular velocity ω. Assuming the radial expansion $\Delta R \ll R$, what is the fractional elongation $\frac{\Delta R}{R}$ of the ring?

A. $\frac{\rho\omega^2 R^2}{E}$

B. $\frac{\rho\omega^2 R^2}{2E}$

C. $\frac{2\pi\rho\omega^2 R^2}{E}$

D. $\frac{\rho\omega^2 R^2}{4E}$

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18 Which of the following physical quantities shares identical SI base units with the Young modulus E?

A. Energy per unit time (Power)

B. Energy per unit area (Surface Energy)

C. Energy per unit volume (Energy Density)

D. Momentum per unit volume

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19 An elastic uniform cylindrical wire is stretched by a hanging weight until its extension is e. If the mass of the hanging weight is strictly doubled, and simultaneously the wire is replaced by one of the identical material and length but twice the diameter, what will be the new extension e'?

A. $e' = 4e$

B. $e' = 2e$

C. $e' = \frac{1}{2}e$

D. $e' = \frac{1}{4}e$

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20 A solid cube of copper is subjected to a uniform hydrostatic pressure ΔP. The bulk modulus of copper is defined as $K = -V\frac{\Delta P}{\Delta V}$. If each linear side length s of the cube contracts by a small fraction $\beta = -\frac{\Delta s}{s}$, what is the corresponding fractional volumetric compression $-\frac{\Delta V}{V}$?

A. β

B. β^3

C. 3β

D. $3\beta + 3\beta^2 + \beta^3 \approx 2\beta$

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21 During an accurate Cavendish-style torsion balance calibration, a vertical cylindrical wire of length L, radius r, and shear modulus G is twisted through an angle θ at its free end. The restoring torque is $\tau = C\theta$. How does the torsional rigidity constant C scale with the radius r?

A. $C \propto r$

B. $C \propto r^2$

C. $C \propto r^3$

D. $C \propto r^4$

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22 A non-uniform solid rod of length L has a linear mass density that increases linearly from $\lambda(0) = \lambda_0$ at $x = 0$ to $\lambda(L) = 3\lambda_0$ at $x = L$. What is the total mass M of the rod?

A. $M = \lambda_0 L$

B. $M = \frac{3}{2}\lambda_0 L$

C. $M = 2\lambda_0 L$

D. $M = 4\lambda_0 L$

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23 When an engineer designs a cantilever bridge truss against plastic failure, which property on the engineering stress-strain curve indicates the maximum tensile stress the material can sustain before microscopic necking and ultimate failure occur?

A. Proportional limit

B. Elastic limit

C. Yield strength

D. Ultimate tensile strength (UTS)

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24 (BPhO Problem) A heavy uniform chain of total mass M, length L, and uniform Young modulus E and cross-sectional area A is suspended vertically from a rigid ceiling. What is the total elongation ΔL of the chain caused exclusively by its own distributed weight?

A. $\Delta L = \frac{MgL}{AE}$

B. $\Delta L = \frac{MgL}{2AE}$

C. $\Delta L = \frac{MgL}{3AE}$

D. $\Delta L = \frac{2MgL}{3AE}$

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25 A metallic wire of resting length $2.0\ \mathrm{m}$ and cross-sectional area $1.0 \times 10^{-6}\ \mathrm{m}^2$ has Young modulus $1.0 \times 10^{11}\ \mathrm{Pa}$. It is stretched by an external mechanical force from an extension of $1.0\ \mathrm{mm}$ to $3.0\ \mathrm{mm}$. Calculate the total external mechanical work done during this extension.

A. $0.050\ \mathrm{J}$

B. $0.200\ \mathrm{J}$

C. $0.400\ \mathrm{J}$

D. $0.800\ \mathrm{J}$

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26 A solid sphere floats in equilibrium at the planar interface between two immiscible liquids of densities ρ_1 and ρ_2 (with $\rho_1 < \rho_2$). If exactly 40% of the sphere's volume lies in the upper liquid and 60% in the lower liquid, what is the average bulk density ρ_s of the sphere?

A. $\rho_s = 0.50\rho_1 + 0.50\rho_2$

B. $\rho_s = 0.40\rho_1 + 0.60\rho_2$

C. $\rho_s = 0.60\rho_1 + 0.40\rho_2$

D. $\rho_s = \sqrt{\rho_1\rho_2}$

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27 A spring-mass oscillator has natural angular frequency $\omega_0 = \sqrt{\frac{k}{m}}$. If the original spring is cut exactly into two halves and connected in parallel with the same mass m, what is the new natural angular frequency ω' ?

A. $\omega' = \omega_0$

B. $\omega' = \sqrt{2}\omega_0$

C. $\omega' = 2\omega_0$

D. $\omega' = 4\omega_0$

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28 (BPhO Extension) An isotropic solid bar of length L, cross-sectional area A, Young modulus E, and coefficient of linear expansion α is clamped securely between two rigid immovable walls at temperature T. If the temperature drops by ΔT, tensile thermal stress develops. What is the total elastic strain energy stored in the contracted bar?

A. $U = \frac{1}{2} ALE\alpha^2(\Delta T)^2$

B. $U = ALE\alpha^2(\Delta T)^2$

C. $U = \frac{1}{2} \frac{AL}{E} \alpha \Delta T$

D. $U = \frac{1}{2} AE\alpha^2(\Delta T)$

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29 Under microscopic investigation, what is the primary crystalline mechanism responsible for irreversible plastic deformation in ductile metals subjected to stresses exceeding the critical resolved shear stress?

A. Symmetric stretching of interatomic covalent bonds without bond breakage.

B. Motion and propagation of line defects (dislocations) along crystallographic slip planes.

C. Sudden isotropic phase transformation into an amorphous vitreous state.

D. Diffusion of free conduction electrons toward the exterior specimen boundaries.

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30 (BPhO Olympiad Finale) A solid sphere of mass M, radius R, and bulk modulus K is placed under enormous hydrostatic pressure in the deep ocean, such that the pressure increases uniformly by ΔP. Treating the density ρ as nearly uniform, what is the fractional increase in the sphere's density $\frac{\Delta \rho}{\rho}$?

A. $\frac{\Delta \rho}{\rho} = \frac{K}{\Delta P}$

B. $\frac{\Delta \rho}{\rho} = \frac{\Delta P}{K}$

C. $\frac{\Delta \rho}{\rho} = \frac{3\Delta P}{K}$

D. $\frac{\Delta \rho}{\rho} = \frac{(\Delta P)^2}{K^2}$

Comprehensive Answer Key

Verified for Olympiad Assessment

Q1: **B**	Q2: **B**	Q3: **B**	Q4: **B**	Q5: **D**	Q6: **B**	Q7: **B**	Q8: **B**	Q9: **C**	Q10: **C**
Q11: **A**	Q12: **B**	Q13: **C**	Q14: **A**	Q15: **B**	Q16: **A**	Q17: **A**	Q18: **C**	Q19: **C**	Q20: **C**
Q21: **D**	Q22: **C**	Q23: **D**	Q24: **B**	Q25: **B**	Q26: **B**	Q27: **C**	Q28: **A**	Q29: **B**	Q30: **B**